Diff. ① Partially with regard to $x$ we have

$$\frac{dF}{dz} \cdot \frac{\partial z}{\partial x} = \frac{\partial \phi}{\partial x} + i \frac{\partial \psi}{\partial x}$$

$$\frac{dF}{dz} = \frac{\partial \phi}{\partial x} + i \frac{\partial \psi}{\partial x}$$

$z = x + iy$

$\frac{\partial z}{\partial x} = 1, \ \frac{\partial \psi}{\partial x} = -\frac{\partial \phi}{\partial y}$

By - C-R- Equation

$$\frac{dF}{dz} = \frac{\partial \phi}{\partial x} - i \frac{\partial \phi}{\partial y}$$

$$\frac{dF}{dz} = -u + iv$$

which is called the Complex velocity

Let $q$ be the magnitude of the velocity at any Point then

$$q = \left|\frac{dF}{dz}\right| = |-u+iv|$$

$$q = (u^2+v^2)^{1/2}$$

$$q^2 = (u+iv)(u-iv)$$

$$= \frac{dF}{dz} \cdot \frac{d\bar{F}}{d\bar{z}}$$

Where $\bar{F}$ is the complex conjugate. this is known as Complex velocity. the Point where velocity is zero are called stagnation Points

$$\frac{dF}{dz} = 0$$

$$q = - \text{grad}\, \phi$$

—x—x—

$$\frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial x} = \frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x}$$

$$z = x + iy$$

$$\frac{\partial z}{\partial x} = 1$$

$$\frac{\partial w}{\partial z} = \frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x}$$

$$\frac{dw}{dz} = \frac{\partial \phi}{\partial x} - i\frac{\partial \phi}{\partial y} = -u + iv$$

$$\frac{dw}{dz} = -\frac{m}{r}\cos\theta + i\frac{m}{r}\sin\theta$$

$$\frac{dw}{dz} = -\frac{m}{r}(\cos\theta - i\sin\theta)$$

$$\frac{dw}{dz} = -\frac{m}{re^{i\theta}}$$

$$\frac{dw}{dz} = -\frac{m}{r}e^{i\theta} = -\frac{m}{z} \qquad z = re^{i\theta}$$

$$\frac{dw}{dz} = -\frac{m}{z}$$

$$\boxed{w = -m\log z}$$

This represent the complex Potential due to a source at origin

$$w = -m\log(z-a)$$

at a Point a

—x—

By Integrating (2) we have

$$\psi(x,y,t) = -\int u(x,y,t)\,dy + f(x,t)$$

Differentiating with Partially with regd to $x$ we have

$$\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}\left[\int u(x,y,t)\,dy\right] + \frac{\partial}{\partial x} f(x,t)$$

$$\Rightarrow \frac{\partial}{\partial x} f(x,t) = \frac{\partial \psi}{\partial x} + \frac{\partial}{\partial x}\left[\int u(x,y,t)\,dy\right]$$

$$\Rightarrow \frac{\partial}{\partial x} f(x,t) = P(x,y,t) \text{ say}$$

$$\Rightarrow \frac{\partial}{\partial x} f(x,t) = V(x,y,t) + \frac{\partial}{\partial x}\left[\int u(x,y,t)\,dy\right]$$

$$\Rightarrow \frac{\partial}{\partial x} f(x,t) = P(x,y,t)$$

where $P(x,y,t) = V(x,y,t) + \frac{\partial}{\partial x}\left[\int u(x,y,t)\,dy\right]$

Integrating we have

$$f(x,t) = \int P(x,y,t)\,dx + Q(t)$$

where $Q(t)$ is an arbitrary function.

$$P(x,y,t) = V(x,y,t) + \frac{\partial}{\partial x}\left[\int u(x,y,t)\,dy\right]$$

$$\frac{\partial P}{\partial y} = \frac{\partial V}{\partial y} + \frac{\partial}{\partial y}\left[\frac{\partial}{\partial x}\int u(x,y,t)\,dy\right]$$

$$= \frac{\partial V}{\partial y} + \frac{\partial}{\partial x}\left[\frac{\partial}{\partial y}\int u(x,y,t)\,dy\right]$$

$$\frac{\partial P}{\partial y} = \frac{\partial V}{\partial y} + \frac{\partial u}{\partial x} = 0$$

$$\frac{\partial P}{\partial y} = 0$$

$\Rightarrow P$ is a function of $x$ and $t$

$$\psi(x,y,t) = -\int u(x,y,t)\,dy + \int P(x,y,t)\,dx + Q(t)$$

this relation represent that the two system of curves of constant velocity Potential and the Stream function cut each other orthogonally.

$\phi(x,y) =$ constant Represent curves of constant velocity Potential.
And $\psi(x,y) =$ constant represent curves of constant stream function.

—x—

Complex Potential and Complex Velocity:–

Consider

$$F(z) = \phi(x,y) + i\psi(x,y) \quad \text{—— (1)}$$

to be an analytic function of the complex Variable. $z = x+iy$

Differentiating (1) with regard to $x$ & $y$ we have.

$$\frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x} = F'(x+iy) \quad \text{—— (2)}$$

$$\frac{\partial \phi}{\partial y} + i\frac{\partial \psi}{\partial y} = iF'(x+iy) \quad \text{—— (3)}$$

$$\frac{\partial \phi}{\partial y} + i\frac{\partial \psi}{\partial y} = i\left(\frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x}\right)$$

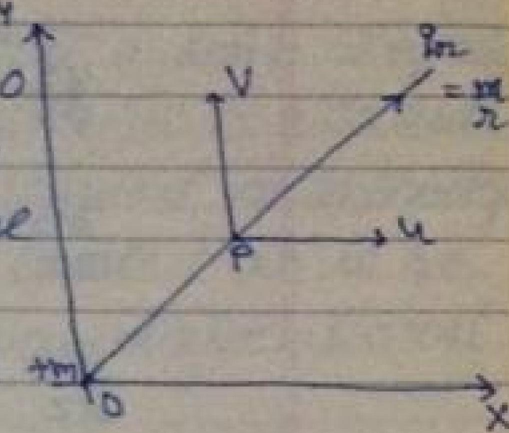
$$\frac{\partial \phi}{\partial y} + i\frac{\partial \psi}{\partial y} = i\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x}$$

$$\Rightarrow \quad \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}, \quad \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}$$

which represent the Cauchy - Riemann Equation.

## Complex Potential of a Source:-

Consider a source of a strength $+m$ at an origin O. Let $q_r$ be the radial velocity at a distance $r$ from the source.




the flux across a circle of radius $= 2\pi r \rho q_r$

Flux across any small curve surrounding the source of strength $+m$

$$= (2\pi m)\rho$$

$$2\pi r \rho q_r = 2\pi m \rho$$

or $\quad r q_r = m$

$$\frac{m}{r} = -\frac{\partial \phi}{\partial r} = -\frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

Small the flux is radial $q_\theta = 0$

$$-\frac{\partial \phi}{\partial r} = -\frac{m}{r}$$

$$\Rightarrow \phi = -m \log r + c$$

where c vanish as $\phi$ and $r$ vanish

$$\Rightarrow \boxed{\phi = -m \log r}$$

Again $\quad -\frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{m}{r}$

$$\Rightarrow \psi = -m\theta$$

constant vanish

as $\theta$ and $\psi$ both vanish

Let $w$ be the complex Potential due to a source of strength $+m$ at the origin

$$w = \phi + i\psi$$

$$\frac{\partial w}{\partial x} = \frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x}$$